NASA TT F-9387

NASA TT F-9387

APPLICATION OF INFORMATION THEORY TO PHOTOGRAPHIC SYSTEMS

D. S. Lebedev

Translation of "O primenenii teorii informatsii k fotograficheskim sistemam"
Zhurnal Nauchnoy i Prikladnoy Fotografii i Kinematografii, Vol. 10,
No. 1, pp. 62-71, 1965

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON JUNE 1965

NASA TT F-9387

APPLICATION OF INFORMATION THEORY TO PHOTOGRAPHIC SYSTEMS

D. S. Lebedev

Table of Contents /62*

*Numbers given in the margin indicate the pagination in the original foreign text.

1. Introduction

Although the performance of modern photographic systems is extremely high, the problem of their further improvement remains quite acute. Since the purpose of photographic systems is to remember the information which is contained in images, it is being hoped that the application of the information theory will make it possible to move substantially in the direction of achieving optimum characteristics, for example, achieving the maximum sensitivity and the best image properties.

The first works which considered photographic systems from the position of information theory appeared very shortly after the theory itself was formulated. At the present time, there is quite a large number of such works. Unfortunately, known works (whose survey is far from complete) do not make it possible for us to judge the soundness of the hopes which have been placed on information theory. Furthermore, the methods of information theory are not applied with complete accuracy in all of the works.

The purpose of this report is to establish how information theory is associated with the problems of perfecting photographic systems.

2. Basic Concepts of the Theory of Information

Information theory is a scientific discipline which is concerned with the method of transmitting information in a most reliable and economic fashion.

The purpose of information transmission is to reproduce a situation at a given place corresponding to a situation at another place. Thus, in television, an effort is made to reproduce on the screen of the receiver the brightness distribution of objects situated in front of the television camera lens.

In many cases it becomes necessary to memorize information. By memorizing information it is possible to reproduce in the future a situation which has occurred in the past. Such a memory, for example, is achieved with a photographic system in which the distribution of brightness on a photographic picture reproduces the brightness distribution of objects which are photographed. The situation, which is subject to reproduction, is called a communication.

In television photography and in motion picture photography the role of initial communication is played by the optical image of objects produced by an ideal optical system. The purpose of a photographic or television system is to reproduce this image.

The reproduction of a communication is delegated to a recipient. The recipient of an image is a human observer, an interpreter, etc.[1] It is impossible and unnecessary to reproduce communications with absolute accuracy. In /63 information theory a measure is introduced for the correspondence of reproduced communication to the initial communication--the accuracy of reproduction.

In scientific photography, in aerial photography, etc., the concept of "accuracy" is equivalent to the concept of "quality of reproduction," because high quality implies a high degree of correspondence of the image to the original, i.e., high accuracy. The situation is more complex in art photography, where the quality of the image depends on the emotional effect of the latter on an observer. It is doubtful that the application of information theory will be of much use in art photography.

The permissible accuracy of reproduction is determined by the recipient. It depends on the properties of the recipient (for example, on the resolving

[1]Recently the possibility of developing automatic machines for recognizing visual objects has been discussed. Such automatic machines would play the roles of image recipients.

capacity, on inertness and on the contrast sensitivity of vision) and on the purpose of the received communication.

The application of information theory is not concerned with a single communication, but with a large number of communications of a given type, for example, with a large number of possible images. Sometimes this large number is understood in a narrower sense: a large number of various types of portraits, or landscapes or aerial landscapes.

The basic proposition made in information theory consists of the following: a given communication, which is to be transmitted or memorized, is the result of a random selection from a large number of communications. In this case each specific communication is assigned a definite probability that it will be selected. A large number of communications together with the probabilities of their selection form a source of communications.

The transmission or memorization of information is accomplished by a combination of technical means called the communication system or a memory system.[1]

The basic problem of information theory is to establish an optimum match between the source of information and the communication system (memory system).

The criteria of optimality may be different. In some cases we must try to achieve the maximum accuracy of reproduction, while in others we must achieve a minimum consumption of photographic material or a minimum required energy (maximum sensitivity), etc. In order to establish the effective utilization of a given communication system (memory system), i.e., to establish the degree of its match with the source, quantitative characteristics of the information source and of the system are introduced.

[1]The term "memory system" is not in general use.

The source is characterized by the average amount of information contained in the communication--by the entropy.

The communication system (memory system) is characterized by the maximum amount of information which a given system can transmit or memorize. This quantity is known as the capacity of the system.[1] Let us consider briefly how the quantitative measure of information is introduced in the theory.

The concept of the amount of information is closely associated with the concept of uncertainty. Let us consider a certain experiment which has several outcomes. The outcomes, for example, may be the appearance of some number when dice are rolled, the birth of a baby of definite sex, etc. Before the experiment there is uncertainty as to which outcome will take place. In information theory a measure is established for the uncertainty of such experiments--the entropy.

If experiment A has N equally probable outcomes, then the entropy of the experiment H(A) is equal to the logarithm of the number of possible outcomes:

$$H(A) = \log_2 N. \tag{1}$$

The larger the number of outcomes, the greater is the uncertainty.

The selection of the logarithm bases determines the units of uncertainty. Thus, if the logarithm is computed with a base 2, we obtain binary units, which are called bits in foreign literature.

In the case of N outcomes which have feasibility probabilities $P_1, P_2 \ldots P_N$, where $\sum_{i=1}^{N} P_i = 1$, the entropy is given by the following expression:

$$H(A) = - \sum_{i=1}^{N} P_i \log P_i \text{ bits.} \tag{2}$$

[1]The term "carrying capacity" is frequently used when information is transmitted.

It is easy to see that equation (1) is a specific case of (2) when all probabilities are the same, i.e., $P_i = 1/N$, $i = 1, 2, \ldots, N$.

Let us assume that before experiment A has been conducted, the outcome /64 of another experiment B has become known. In this case, generally speaking, the probabilities of the outcomes of experiment A will change, and consequently its entropy will also change.

Let us assume, for example, that experiment A consists of counting the number of developed grains in a given region of a photographic image, while experiment B consists of measuring the exposure of this region. It is clear that the distribution of the number of grains for a given exposure is different than when the exposure is unknown and has a random value.

Let us designate by $P_j(i)$ the probability of the i-th outcome of experiment A, if we know that experiment B had an outcome j. Let r_j be the probability j-th outcome of experiment B which has M outcomes and $\sum_{j=i}^{M} r_j = 1$. Then the average (taking into account the various outcomes of experiment B) uncertainty of experiment A, when we know the outcomes of B, will be expressed by the conditional entropy:

$$H_B(A) = \sum_{j=1}^{M} r_j \left(-\sum_{i=1}^{N} P_j\ (i) \log P_j\ (i) \right) . \tag{3}$$

Information on the results of experiment B can only decrease the uncertainty of experiment A:

$$H_B(A) \leq H(A).$$

The equality $H_B(A) = H(A)$ occurs when experiments A and B are independent.

Let us determine the quantity I(A,B)--the amount of information concerning experiment A contained in the information concerning the outcome of experiment B, in the following manner:

$$I(A,B) = H(A) - H_B(A). \tag{4}$$

In other words, the measure of the quantity of information is the measure of the variation in the uncertainty associated with the obtainment of this information.

Substituting (2) and (3) into (4) and performing some transformations we obtain:

$$I(AB) = \sum_{i=1}^{N} \sum_{j=1}^{M} P(ij) \log \frac{P(ij)}{P_i r_j} . \tag{5}$$

where $P\,(ij) = r_j P_j(i)$ is the combined probability of the i-th outcome of A and the j-th outcome of B (in this case $\sum_i P(ij) = r_j$, $\sum_j P(ij) = P_i$). From expression (5) we obtain the following important result:

$$I(AB) = I(BA).$$

The amount of information in experiment B with respect to experiment A is equal to the amount of information in experiment A with respect to experiment B.

Expression (5) may be generalized for the case when one of the experiments, or both of them, have an infinite number of outcomes.

3. The Amount of Information in Images

As we already stated, a specific image is selected in a random fashion from a large number of images. If a large number of images has a finite number of different images (in a given area), then by indicating which image is selected we can remove the uncertainty completely. Thus, in this case, the average amount of information in images of a given source--the entropy of image source -- coincides with the entropy--the measure of uncertainty in selecting images.

Let us assume, for example, that the source of images produces congratulatory telegram forms and these forms, on the average, are selected with the same frequency. The entropy of such a source is:

$$H = \log_2 N,$$

where N is the number of different forms. Since the number N is equal to several tens (let us say from 32 to 128) we have

$$H = 5 - 7 \text{ bits.}$$

Let us consider another example. Every image can be divided into elements. This is done in polygraphy and phototelegraphy, etc. The size of the element is selected taking into account the resolving power of the eye and of the angle at which the image is observed (we note that the use of a lens in front of the eye means that this angle is increased). For example, in phototelegraphy (facsimile) the size of the element is $0.2 \cdot 0.2$ mm, so that an image of $9 \cdot 12$ cm /65 contains 270,000 such elements.

Now let us assume that the brightness (density) of each element has M graduations of brightness. We have a total of $N = m^s$ images, where s is the number of elements. If all images are considered of equal probability, we have

$$H = \log_2 N = s \log_2 m,$$

substituting $s = 270{,}000$, $m = 32$, we obtain:

$$H = 1{,}350{,}000 \text{ bits.}$$

These examples show that the entropy of the image source is the property of the entire source and not of individual images. The same image may have 5 or $1.5 \cdot 10^6$ bits, depending on the multitude from which it is selected: the multitude of congratulatory telegram forms or the multitude of different combinations of brightness of 270,000 elements.

The examples which we have presented are of a somewhat artificial nature. What is the quantity of information in "natural" images of photographed objects produced by an ideal optical system.

First of all, we note that the number of such stationary images of a given area is finite. Indeed, the image may be described as a distribution of electromagnetic field intensity in the plane of the photographic layer. A unit area of this plane contains only a finite number of independent degrees of freedom, determined by the aperture of an ideal lens[1] (for example, see reference 1). Thus, the image is determined by the distribution of the electromagnetic field energy over these degrees of freedom. If we take into account the quantum nature of the electromagnetic field, we may find the number (and probability) of various distributions, i.e., we can find the entropy of the multitude of optical images. The evaluation of this quantity was carried out by Fellgett, Linfoot and by others (refs. 2-6).

We shall not discuss these works because the quantity of information which can be extracted by the recipient from images is several orders of magnitude less than that given by any of such evaluations.

The recipient cannot distinguish between many images which represent different energy distributions according to the degrees of freedom of the image: the visual system has a threshold of absolute and contrast sensitivity, and it is incapable of differentiating colors composed of different wave-length radiations, but having the same color coordinates, etc.

If all indistinguishable images are grouped and if instead of any member of the group we always transmit the same image--"representative image"--the number of possible images and, consequently, the entropy are decreased. A further decrease in entropy will occur if we combine in a group not only the indistinguishable images, but also images which are slightly distinguishable. In this case the replacement of the initial image by an image which differs slightly from it will mean that the quality of image reproduction will deteriorate. It is clear that the larger the permissible deterioration, the smaller is the entropy of the

[1]The aberration of a real lens decreases the number of degrees of freedom.

image force. Therefore, the quantity of information in an image is associated with the assigned quality of this image.

In order to compute the entropy of the image source from the point of view of the recipient, it is necessary to introduce a criterion of accuracy, i.e., for each pair of images $B(x, y)$ and $B'(x, y)$ we must assign a corresponding number $\rho(B(x, y), B'(x, y))$, which shows the "closeness" or resemblance of the images.

At the present time the form of the functional $\rho(B(x, y), B'(x, y)$, is not known accurately; however, an approximate evaluation of the entropy can be made. Extending the results obtained in reference 7, we can evaluate the entropy as exceeding 1.6 bits per unit image element (this, for example, gives us 400,000 bits in an image $9 \cdot 12$ cm when the element size is $0.02 \cdot 0.02$ cm).

Generally speaking, the form of the functional $\rho(B, B')$ depends not only on the physiological properties of the recipient, but also on the use which is to be made of the image. If, for example, the purpose of the recipient is to distinguish between a female portrait and a male portrait, the multitude of images will now consist of only two elements: male portrait and female portrait.

In computing the amount of information, all different male portraits must be considered as indistinguishable, because, even though the recipient can distinguish them, he must not do so.

As a matter of fact, within the framework of information theory applied to photographic systems, such degenerate cases are not encountered.

4. The Capacity of the Photographic System

The capacity of a photographic system is the maximum amount of information which can be memorized by a given system. The process of memorizing information consists of the system entering a stable state under the action of specific information. The larger the number of stable states which a memory /66 system has, the greater is the number of communications which it is capable of memorizing. When we know the state of the system, we can reproduce the communication, if we know the rule which associates different communications with different states.

The reflection of the communications in the state of the system is called coding, and the reverse process is called decoding.

The state of the photographic system may be described by the distribution of the blackening of a photographic layer--$D(x, y)$.

Coding in photography consists of projecting the optical image described by the brightness distribution $B(x, y)$ on a photographic layer. Decoding, i.e., the reproduction of brightness distribution $B'(x, y)$ as a function of the systems state may be accomplished, for example, by projecting the image $B'(x, y)$ on a screen.

The simplicity and "naturalness" of these transformations $B(x, y) \rightarrow D(x, y)$ and $D(x, y) \rightarrow B'(x, y)$, at first glance, indicates that the introduction of the concepts "coding" and "decoding" in photography is unnecessary.

However, as we shall show later, the transformation from natural operations of projection to more complex methods of coding and decoding may substantially increase the effectiveness of applying a photographic system.

Let us now consider in more detail the operation of a photographic system. The image of the objects to be memorized is projected by an optical system on a light-sensitive layer. Let us divide the photographic layer into elements corresponding to independent degrees of freedom of the optical image. A certain quantity of photons falls on each element of the layer, and the average number of these (i.e., the energy per degree of freedom) is determined by the initial image. The action of photons is responsible for the presence of metallic silver grains in this element after development.

The number of these grains is associated statistically with the number of photons: the average number of developed grains is a function of the number of photons. The actual number of grains fluctuates around an average value. The number of developed grains determines the density of the element and consequently the brightness of the reproduced image.

Due to the presence of fluctuations, information on the number of developed grains leaves some uncertainty in the energy of the element of the initial image; however, this uncertainty is less than the initial uncertainty. In other words, the value of the number of grains contains certain information about the energy (or to the brightness, which is proportional to it) of the element in the initial image.

In accordance with the generally accepted theory of the photographic process, the number of developed grains in a given element is a random quantity which satisfies Poisson's distribution law:

$$P_a(n) = \frac{a^n}{n!}e^{-a}. \qquad (6)$$

Here $P_a(n)$ is the probability that the number of developed grains is equal to n, while a is the average number of developed grains. If a_M is the average number of light sensitive grains, while $\pi(E)$ is the probability that an individual grain is developed after it is subjected to the action of light with energy E, then

$$a = a_M\pi(E). \qquad (7)$$

The dependence of quantity a on the energy E may be established experimentally: directly or by superimposing two experimental curves D(a)--the variation in density as a function of the number of grains, and D(E)--the characteristic curve of the photographic material.

The energy of the image element may assume any values. We must compute $I(E, n) \rightarrow$ the amount of information about this quantity contained in our knowledge of another random quantity--the number of developed grains n.

Since according to (7) the quantity E gives a single value for a, which is therefore also a random quantity, we have

$$I(E, n) = I(a, n).$$

It is more convenient to compute the amount of information $I(a, n)$ than it is to compute $I(E, n)$.

When the energy E changes, the average number of developed grains varies in the limits a_0, a_M. The quantity a_0 corresponds to the density of the fog when E is small. The quantity a_M is achieved when the energy E is very large, so that $\pi(E) = 1$.

If we know p(E)--the distribution density of the random quantity E, we may find p(a)--the distribution density of the random quantity a.

Now we can derive an expression for $I(a, n)$

$$I(a, n) = \int_{a_0}^{a_M} p(a) \left\{ \sum_{n=0}^{\infty} P_a(n) \log \frac{P_a(n)}{P(n)} \right\} da, \tag{8}$$

where $P(n) = \int_{a_0}^{a_M} p(a)P_a(n)da$.

This quantity of information corresponds to the average energy of the image element /67

$$E = \int Ep(E)dE. \tag{9}$$

Thus the amount of information $I(a, n)$ and the average energy E is determined by the distribution p(E), i.e., by the statistical properties of the image source.

Let us select the distribution p(E) so that expression (8) has a maximum value. The maximum value of (8) is the capacity of the photographic layer of element. Let us designate this quantity by C_0

$$C_0 = \max_{p(E)} I(a, n) = \max_{p(a)} I(a, n). \qquad (10)$$

Substituting the optimum distributions into (9), we find $\overline{E}_0$.

The ratio $\overline{E}_0/C_0$ is the quantity of energy dissipated for one binary unit of the memorized information, when C_0 is a maximum.

It is interesting to solve this problem: to select p(E) in such way that this ratio is minimized. After we find this distribution and substitute it into (9), and substitute the corresponding distribution p(a) into (8), we find quantities $\overline{E}_1$ and C_1.

It is obvious that $\overline{E}_1/C_1 \geq \overline{E}_0/C_0$, $C_1 \leq C_0$.

If the elements of the photographic layer were independent, then the capacity of the photographic system would be obtained by multiplying C_0 by the number of elements. However, the elements of the layer, unlike the elements of an optical image produced by an ideal optical system (without aberration), are not independent. Due to the presence of aberration and the scattering of light in a photographic layer, the illumination of a given element depends on the illumination of the neighboring elements.

There are two approaches for an approximate solution to the problem of computing the capacity of a photographic system. In the first place, we may assume that the elements of the photographic layer are independent, and that their value is equal to the area of the circle of dispersion[1] or is determined by the

[1]The area of the circle of dispersion is to some extent conditional, because frequently the dispersion function is approximated by functions which extend to infinity—by an exponential function, Gauss function, etc.

resolution of the photographic system. (This approach to the evaluation of the number of independent elements is contained in reference 8.)

Another approach is to use the frequency description of the image, i.e., to replace the brightness distribution (energy) along the coordinates by the distribution of the intensity of the components of various spatial frequencies. It is obvious that this approach is suitable for optical images. However, the application of the frequency description of the state of a photographic material encounters serious difficulties, produced by the nonlinearity of the photographic process.

Linfoot and others (refs. 9 and 10) have attempted to bypass these difficulties by considering images of low contrast. In this way correct results are obtained for the amount of information memorized by a photographic system, when it operates with sources of low contrast images. However, this amount of information is not the same as the capacity--the maximum amount of information during the best utilization of the system.

Jones (ref. 11) made an effort to expand the Linfoot approach to an image of high contrast. However, his results, as acknowledged by the author himself, are not entirely correct, although apparently the evaluation of the capacity is of the proper order.

If we know the magnitude of the capacity of the memory system and the magnitude of the entropy of the information source, it is possible to determine the conditions under which the given system is suitable for memorizing information from a given source, and to determine the effectiveness of the memory system.

This question is answered by a basic theorem of information theory proposed by Shannon (ref. 12), which proposes the following.

If a source with a given accuracy ϵ has an entropy H_ϵ, we can code the information of the source and transmit it through a system with capacity C, with a reproduction accuracy arbitrarily close to ϵ, if only $H_\epsilon \leq C$. This is impossible if $H_\epsilon > C$.

Let us clarify the meaning of the theorem by using the following, somewhat artificial example. Let us assume that the source of images has M equally probable distinguishable recipients of information, and let us assume that the system has N states. Then

$$H = \log_2 M \text{ bits and } C = \log_2 N \text{ bits.}$$

The second part of the theorem, which maintains that $H \leq C$ is a necessary condition for transmission, now appears obvious: when $H > C$ (i.e., when $M > N$), the number of system states will not be sufficient.

The proposition that condition $H \leq C$ is sufficient is not obvious. Indeed, it does not follow from $M \leq N$ that a method can always be found to juxtapose each communication to each state. The fact that this is actually possible /68 is demonstrated by the proof of the theorem.

(The proof of the basic theorem may be found in reference 12 or in any textbook on information theory.)

If an image contains H bits of information, and if its memorization with a given coding method requires a photographic system with a capacity of C bits, we can say that the effectiveness of utilizing the system under these methods of coding is H/C. If the coding method is perfected, the effectiveness can be increased. The basic theory maintains that there are coding methods for which the effectiveness can be made to approach unity.

The effectiveness of utilizing the memory system depends on the degree to which the statistical properties of the information source correspond to those properties which provide for a maximum amount of memorized information.

By changing the statistical properties of the multitude of communications through coding, it is possible to achieve a better matching of these properties to the requirements of the system, i.e., it is possible to increase the effectiveness.

Different memory systems may require different statistical properties of the matched source. Therefore, if the same method of coding information is used for the same source to match different memory systems, it may turn out that a system with a high capacity may memorize less information than a system with a smaller capacity. This discussion again shows how careful we must be in evaluating, for example, the quality of photographic materials with respect to their capacity.

The higher the effectiveness of utilizing a photographic system, the lower its required area of photographic material for memorizing images with a given amount of information. The minimum area is determined by the following simple consideration.

Let us assume that the area of an independent element (which is of the same order of magnitude as the dispersion circle) is equal to σ, and its capacity is C_0 bits. If the image contains H bits of information, the minimum area (when the effectiveness is equal to unity, i.e., for the case of ideal coding), is equal to:

$$S_{min} = \sigma H/C_0.$$

By going from one coding method to another more effective one, it is possible not only to decrease the area occupied by the image, but to improve the quality of the image. (Let us recall that when the quality is higher, the entropy of the image is also higher.)

The concepts of information theory may be used to investigate the maximum sensitivity of photographic systems.

The transmission and the memorization of information, according to the Brillouin principle (ref. 15), is associated with an expenditure of energy. During photographing the necessary energy is provided by the radiation from the photographed objects. As a measure of the sensitivity of a photographic system it is natural to consider the energy necessary for remembering a definite amount of information, for example, one bit. In the latter case the sensitivity is given by the ratio $\overline{E}/I(a, n)$.

The maximum sensitivity is equal to $\overline{E}_1/C_1$. Since $C_1 \leq C_0$, operation with maximum sensitivity may be accompanied by a loss in the utilized area of photographic material.

Let us consider the data presented in the works of Jones (ref. 11). We see that for every bit of memorized information the photographic system expends an energy of 8,000-30,000 photons. However, it follows from thermodynamic considerations associated with the Brillouin principle (refs. 13 and 14) that in principle it is sufficient to have energy of the order of one photon. The relatively low sensitivity is the principle shortcoming of the photographic system.

The possiblities of coding by means of photographic methods are extremely limited. Of course, another approach can be used, for example, electron-optical coding; however, this will make meaningless the formulation of the problem on the possibility of using the methods of information theory to perfect photographic systems.

In connection with the difficulties of optimum coding in photography, some investigors (refs. 15 and 16) examined the true capacity of a photographic system--its capacity to memorize digital information under specific conditions of introducing and reading this information.

5. The Quality of the Image

For a communications engineer it is rational to apply information theory, because this theory makes it possible to evaluate the effectiveness of using a communications system. If the effectiveness is known, it is possible to have a basis for selecting the method of coding and decoding with any degree of complexity. In photography, as we have already stated, the coding possibilities are limited, which decreases the value of knowing the magnitude of effectiveness.

However, the ideas of information theory in the broad sense may be used effectively to develop optimum photographic systems.

The works of Elias (ref. 17) and Fellgett (ref. 18) appeared immedi- /69
ately after Shannon developed the theory of information. These works underline the analogy between electrical communications systems and optical and photographic systems. Because this analogy was recognized, frequency concepts--spectrum of images and contrast--frequency characteristics of optical and photographic layers, were introduced into photography and optics. To describe density fluctuations, which appear as a grain structure, the theory of random processes was applied (specifically the correlation theory of stationary processes), which treated these fluctuations like electrical noise. This description of photographic and optical systems produced a new approach to the understanding of image quality.

In optics the only form of distortion is the attenuation of higher spatial frequencies due to diffraction and aberration. The greater this attenuation, the smaller is the resolution, image definition, etc.

In television and in photography the frequency distortions are supplemented by distortions of contrast--nonlinear distortions and fluctuating noise--graininess, noise, etc.

Many efforts were made to associate the quality of optical, television or photographic images with the characteristics of the corresponding systems (refs. 19-23). None of these efforts, as yet, have achieved unconditional success because the selection of the accuracy criterion (quality) has been unsuccessful.

The problem of evaluating the quality of an image must be formulated in the following manner: there is a criterion for the accuracy of the image--the functional $\rho(B,B')$, which assigns some number to two images B and B' for characterizing their closeness or resemblance. Usually the form of $\rho(B,B')$ is selected in such a way that when the number is large, the resemblance is small. Then $\rho(B,B')$ has the sense of "distance" between images.

The form of $\rho(B,B')$ is determined by the properties of the recipient and by the purpose of the image.

The form of the functional $\rho(B,B')$ may be established from biophysical and psychological investigations. At the present time this problem is not completely solved.

Investigations in the theory of information frequently use the root-mean-square criterion of accuracy (ref. 12).

In this case the functional $\rho(B,B')$ is selected in the following manner

$$\rho(B,\ B') = \int \int [B(x,\ y) - B'(x,\ y)]^2 dx\, dy. \tag{11}$$

Integration in equation (11) is carried out over the entire area of the image.

Unfortunately, this criterion does not correspond very well to the properties of the human visual system. This is evident if we consider the image B' (x, y), which differs from B(x, y) in almost each element by a magnitude less than the contrast threshold of vision. In this case the distortions will not be noticeable, even though the value of (11) may achieve a substantial magnitude ρ_0.

At the same time, distortions in a few elements are such that $\rho(B,B') = \rho_0$ will be clearly noticed as foreign points.

Let us consider in detail the information criterion of accuracy. The authors of this criterion (for example, ref. 23) reason in the following way. It is assumed that the quantity of information, which is memorized or transmitted to some system, when operating with a given source, is I_1 and that transmitted to another system is equal to I_2. If $I_1 > I_2$, then the first system provides for a better quality of the image than the second. In this discussion it is assumed that there is a simple relationship between the quantity of information about the image and its quality.

Let us assume that we have a system that transforms an image. If this transformation is reversible, the distortion of the image due to the transformation may be compensated in principle. In this case the amount of information memorized by the system reachs a maximum value--the entropy of the image source. The same value is achieved with an ideal system without distortion. Thus, two systems without distortion and with any amount of distortion, obtained as a result of reversible transformation, provide for the same amount of information.

If irreversible transformations take place in a system, the quantity of memorized information will be less than the entropy of the initial image. However, in this case too, we cannot establish an relationship between this quantity and the quality of the image. Due to irreversible transformations, the individual images are combined into groups which cannot be distinguished. The larger these groups, the more pronounced is the decrease in the amount of information.

However, the quality of the images is effected not only by the magnitude of the indistinguishable groups, but more so by their "form." If, for example, some system combines information into groups just like the recipient, the distortions will not be noticed at all. However, images which appear substantially different for the recipient may be combined in groups--this will produce a substantial deterioration in quality.

In information theory it is a practice to introduce the criterion of reproduction accuracy obtained from biophysical (or psychological, etc.) investigations and then to compute the amount of information in the image. Of course, in this case, the amount of information increases with higher requirements for quality. The introduction of the "information" criterion of accuracy is an /70 effort to place the problem "from its feet on its head."

Let us assume that it has been possible to introduce a suitable criterion of accuracy $\rho(B,B')$. In this case the problem of evaluating the quality of the image is solved automatically. Let us assume that the initial image is $B(x, y)$. If we know the characteristics of the system, we can find the transformed image $B'(x, y)$. We compute $\rho(B,B')$--this is a measure of distortion from the point of view of the recipient. The quality of the image provided by the system must be judged not on the basis of one image, but on the basis of a multitude of images. Then the quality is conveniently described by the average distance

$$\bar{\rho} = \sum P_i \rho(B_i, B_i'),$$

where P_i is the probability of some image B_i.

The determination of the accuracy criterion adequate for evaluating the resemblance of the images by the recipient will make it possible to solve a series of important practical problems of the following type.

The criterion of accuracy is given. It is required to select a photographic material which will provide for the required accuracy with a minimum average energy of the projected image or a maximum accuracy with a given energy.

This problem may include the problem of selecting the optimum size of the image on a photographic layer. (This problem for a specific criterion was considered by Fisher, ref. 24.)

We note that this problem is not, strictly speaking, a problem of information theory, because its solution does not require the calculation of the amount of information.

6. Conclusions

1. In describing the properties of photographic systems, it is convenient to use an analogy between these systems and other systems for transmitting and storing information (in particular electric communications systems). This analogy follows from the general concepts of information theory.

2. The development of optimum photographic systems is retarded by the absence of an accuracy criterion for the reproduction of images, i.e., the resemblance criterion of the image from the point of view of the recipient. Such a criterion must be determined by using biophysical methods of psychological investigations, and not by the methods of information theory.

3. The concepts of the information theory--quantity of information, entropy, capacity--have a rather limited application to photographic systems. This is explained by the fact that these concepts are used to measure the effectiveness of coding and decoding, while the achievement of these operations by purely photographic methods is almost excluded.

REFERENCES

1. Maroshal', A. and Franson, N. Structure of the Optical Image (Struktura opticheskogo izobrazheniya). Izd-vo "Mir," 1964.

2. Fellgett, P. and Linfoot, E. Philos. Trans. Roy. Soc. London A, 247, 369, 1955.

3. Linfoot, E. and Black. Proc. Roy. Soc. A, 239, 522, 1957.

4. Linfoot, E. J. Opt. Soc. America, 45, 808, 1955.

5. Blanc-Lapierre. Optica acta, 2, 1, 1955.

6. Toraldo di Francia, G. Optica acta, 2, 5, 1955.

7. Lebedev, D. G. and Lebedev, D. S. Vestn. AN SSSR, 11, 1964.

8. Lyalikov, K. S. and Ivkina, Ye. G. The Entropy of the Aerial Photograph (Entropiya aerosnimka). Uspekhi Nauchn. Fotogr., 10, 1964.

9. Linfoot, E. Photogr. Sc., 7, No. 6, 1959; 9, No. 3, 1961.

10. Shaw, R. Photogr. Sci. Engng, 6, 281, 1962.

11. Jones, R. C. J. Opt. Soc. America, 51, 1159, 1961.

12. Shannon, K. The Mathematical Theory of Communication (Matematicheskaya teoriya svyazi). In collected works: Works on the Theory of Information and Cybernetics, Izd-vo. in. lit-ry, 1963.

13. Brillouin, L. Science and the Theory of Information (Nauka i teoriya informatsii). Fizmatgiz, 1960.

14. Lebedev, D. S. and Levitin, L. B. The Transmission of Information by the Electromagnetic Field (Perenos informatsii elektromagnitnym polem). Problems of Information Transmission, No. 16, 1964.

15. Eyer, J. Photogr. Sci. Engng., No. 2, 1962.

16. Altman and Zweig. Photgr. Sci. Engng., 7, No. 3, 1963.

17. Elias, P. J. Opt. Soc. America, 43, No. 4, 1953.

18. Fellgett, P. J. Opt. Sci. America, 43, No. 4, 1953.

19. Selwyn, E. The Theory of the Resolving Power (Teoriya razreshayushchey sily). In collected works: Evaluation of the Property of Optical Images, Geodezizdat, 1959.

20. Selwyn, E. J. Photogr. Sci., 7, No. 6, 1959.

21. Schade, O. The New System for Determining and Measuring the Definition of an Image (Novaya sistema opredeleniya i izmereniya chetkosti izobrazheniya). In collected work: Evaluation of the Property of Optical Images, Geodezizdat, 1959.

22. Blaschke, W. J. Photogr. Sci., 7, No. 6, 1959.

23. Gurevich, S. B. The Effectiveness and Sensitivity of Television Systems (Effektivnost' i chuvstvitel'nost' televizionnykh sistem). Izd-vo "Energiya," 1964.

24. Fisher, M. J. Photogr. Sci., 10, 65, 1962.